UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

OPTIMAL LOCATION OF CAR WRECK ADJUSTERS

POR

LUIS ALBERTO MALTOS ORTEGA

COMO REQUISITO PARCIAL PARA OBTENER EL GRADO DE MAESTRÍA EN CIENCIAS EN INGENIERÍA DE SISTEMAS

JUNIO, 2016

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SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO

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Los miembros del Comité de Tesis recomendamos que la Tesis "Optimal Location of Car Wreck Adjusters", realizada por el alumno Luis Alberto Maltos Ortega, con número de matrícula 1390200, sea aceptada para su defensa como requisito parcial para obtener el grado de Maestría en Ciencias en Ingeniería de Sistemas.

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San Nicolás de los Garza, Nuevo León, Junio 2016

To my wife Alejandra

to my children they are the reason to beat.

Contents

List of Figures

List of Tables

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Vita

Luis was born in San Nicolas de los Garza, Nuevo León, México on November 25, 1989, the fourth of five sons of Luis Alberto Maltos Muzquiz and Ruth Ortega Pecina. He received a degree in mathematics from Universidad Autónoma de Nuevo León in 2011. He worked as consultant, until 2014, and analyzed mathematical models and solution method. In the begining of the same year he began to study a Master of Science in Systems Engineering.

Abstract

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Candidato para obtener el grado de Maestría en Ciencias en Ingeniería de Sistemas.

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Título del estudio: Optimal Location of Car Wreck Adjusters.

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Problem Description: When a traffic accident occurs in cities with a large traffic flow the roads surrounding the crash site are affected by traffic congestion. In some countries, such as Mexico, even small accidents are troublesome due to the fact that a claim adjuster from the car insurance company must arrive to the site and document the accident before the vehicle may be removed as required by law. Thus, in this particular setting, the location of the adjusters becomes a key factor in providing timely service.

Objectives and method of study: The propouses of this thesis is to

- Provide quantitative tools for scientific support for the optimal location of car wreck adjusters.

- Develop adequate mathematical models for representing some of the important company concerns. Design and develop efficient solution techniques for handling real-world instances of the problem.

- Assess the quality and value of the proposed techniques based on an appropriate experimental design.

Contributions and Conclusions: The most important contributions of this work are listed below:

- With the aim of minimizing average response time, two integer programming models are introduced.
- The second model make special considerations to reduce the number of variables.
- An scatter search heuristic was designed, built and tested over a wide set of instances with very good results.

Firma del asesor: ______________________________
Dr. Roger Z. Ríos Mercado

Chapter 1

Introduction

The main idea behind this thesis is to develop mathematical models to improve the service offered by car insurance agents. The goal is to determine the number of adjusters required to perform the service, within the desired locations, to help them arrive to accident sites sooner.

1.1 Problem Statement

Car insurance companies in countries such as Mexico faces the daily issue of how to locate their insurance agents (adjusters), in such a way that they provide the best possible service to their customers. Typically the quality of service is measured by how quickly the adjusters arrive to the places where an incident has occurred.

This problem can be seen as an emergency location system, in which it is desired to "cover" the territory under study (city intersections) by a fixed number of adjusters so as to reduce the average time of arrival to accidents. An issue that makes the problem even more complex is the fact that accidents occur randomly and decisions on where to place the adjusters must be made before accidents occur.

1.2 Background

The problem of locating car insurance agents is a practical problem that has not been studied before to the best of our knowledge. This comes from the fact that in other countries, when an accident occurs cars drivers are allowed to move their cars from the accident site if this obstructs traffic. Unfortunately, in many developing countries, insurance agencies ask their insurees not to move the car until an adjuster has arrived.

Nonetheless, there are many related location problems that look at similar problems. For example the case of ambulance location problems, for instance, where emergency services (ambulances) must be located in such a way that ambulances arrive promptly at the site of the accident. A main difference between our problem and the one of an emergency service location is that in those problems time is a matter of life-or-death, and in the case of adjusters location it is not necessarily so. This resulting models look then at different objective function and constraints. In Chapter 2, related problems are discussed.

1.3 Motivation

When a car accident occurs, traffic congestion starts to pile up. This is because customers are not allowed to move their vehicles until the adjuster arrives. The adjuster must record and determine the causes of the accident in order to move the car from the accident area and restore the flow. Needless to say, an early arrival means everything to everyone. Customers wait less, and traffic jams are cleared faster when adjuster arrive promptly.

Given that accidents occur randomly, the idea of this research is to use stochastic location models to derive efficient policies based on historical data that can be implemented by car insurance companies

1.4 Objectives

The main objective of this research are:

- Provide quantitative tools for scientific support for optimal location of car wreck adjusters.
- Develop or use adequate mathematical models for representing some of the important company concerns.
- Design and develop efficient solution techniques for handling real-world instances of the problem.
- Assess the quality and value of the proposed techniques based on an appropriate experimental design.

1.5 Organization

This thesis is organized as follows: Chapter 2 presents a brief literature review of divers approaches of Emergency Service Systems (ESS), starting with the simple deterministic models and ending with the hypercube model and simulation models. The problem statement and two proposed models are presented in Chapter 3. Chapter 4 describes the heuristic method proposed and their components. Chapter 5 contains the experiments made to the models, and to the heuristics to tune them. Conclusions, contributions, and directions for future research are highlighted in Chapter 6. Additionally, Appendix A contains a summary of how to approximate the hypercube results, and together with Appendix B completes the explanation of how to estimate the response times of a solution.

Chapter 2

Related Work

Work in emergency vehicle base location has generally involved the use of three approaches: queueing, mathematical programming, and simulation.

In this chapter, we review the most relevant works in each of these approaches.

2.1 Mathematical Programming

Location models are classified in two main categories, deterministic and probabilistic. Deterministic models are typically used at the planning stage and ignore stochastic considerations regarding the availability of servers or the distribution of the demand. Probabilistic models reflect the fact that vehicles operate as servers in a queuing system and are not always available to answer a call; this models permit a more accurate planning of Emergency Service Systems (ESSs) at the strategic level. They were initially developed based on the assumptions that servers are independent and do not cooperate, which is not realistic in practice.

The location literature is very extense, therefore we focus the discussion in this chapter on models that are more typically used in the location of emergency services. As stated before, no previous work on location of car wreck adjusters exist to the best of my knowledge; however, there are many common elements between our problem and the problem of location of emergency services.

2.1.1 P-Median Models

The first approaches incorporate median type objectives, the aim of this problem is Examples of such models are the deterministic p-median of Hakimi [11], the stochastic p-median and the vector assignment p-median of Church and Weaver [22, 23] and the capacitated p-median of Schilling and Pirkul [17]

The strength of these models is that optimal solution procedures have been developed and can accommodate practical problems. Weaknesses include assumptions such as noncooperation between vehicles, the probability of each system state is known and the fraction of call served by the closest, second closest, etc. is known for each zone.

2.1.2 Covering models

In the location set covering model (LSCM) introduced Toregas et al. [21] the aim is to minimize the number of ambulances needed to cover all demand points. This model ignores several aspects of real-life problems, the most important probably being that once an ambulance is dispatched, some demand points are no longer covered. However, the authors provide a lower bound on the number of ambulances required to ensure full coverage.

The maximal covering location problem (MCLP) originally proposed by Church and ReVelle [5] is an alternative approach proposed to overcome some of the shortcomings of the LSCM. In the MCLP the objective is to maximize population coverage subject to limited ambulance availability.

A limitation of the deterministic models is that they assume that servers are available when requested, which is not always true in practical situations. Congestion in emergency services, which may cause the unavailability of servers located within the critical distance when a call is placed, leads to the development of a second generation of location covering models focused on additional coverage.

The definition of probabilistic location models for planning these systems is a natural extension of their deterministic equivalents, the location models with covering constraints. The notion of coverage implies the definition of a service distance (time), which is the critical distance (time) beyond which a demand area is considered not covered. A demand area is therefore considered covered if it is within a predefined critical distance (say D) from at least one of the existing facilities.

The Maximum Expected Covering Location Problem (MEXCLP) defined by Daskin [6] whose objective is to maximize the expected coverage of all demand areas under consideration, assume that servers operate independently and that all servers have the same busy probability (workload) ρ, allowings that more than one server be situated in any given location. Daskin et al. [7], assume that travel times are deterministic and coverage is an "all-or-nothing" property.

ReVelle et al. [18] proposes two variations for the Maximum Availability Location Problem (MALP) locate p servers in suchs a way as to minimize the population which will find a server available within α reliability the firstone assume, like Daskin, that each server has the same busy probability, and predetermine the number of times a demand point needs to be covered. The other, allow busy fractions to be different in the various sections of a region under consideration (but not for each server to be located)

These models emphasize the importance of additional coverage for the demand areas, given the possibility that in congested systems the first server, possibly the only server in a particular coverage area, might not be available when requested. Gendreau et al. [8] proposes a model with double coverage, uses two radious r_1, and r_2 ($r_2 > r_1$), to locate p ambulances, such that all the demand must be covered by an ambulance located within r_2 time units, and, a proportion α of the demand must also be within r_1 units of an ambulance, which may or may not be the same ambulance that covers this customer within r_2 time units. Note that a feasible solution may not exist if the parameters r_1, r_2 and α are too restrictive.

2.2 Queueing Models

The hypercube model (denoted Hypercube) and the hypercube approximation (denoted A-Hypercube). developed by Larson [15, 16] are the most well known queueing approaches. These are not an optimization models; they are only a descriptive models that permits the analysis of scenarios. Both models estimate system operating characteristics that are used to evaluate a series of objectives. They can evaluate cooperation between vehicles, their weaknesses, include

- Assumptions of an exponentially distributed service time.
- Computational difficulties for problems with many vehicles.
- Require that service time be solely vehicle-dependent rather than call location-dependent.

The computational problems are remediated in the A-Hypercube by approximating the vehicle busy probabilities by solving a system of nonlinear equations whose size depends on the number of vehicles.

Optimization models for locating Emergency Medical Services (EMS) that use Hypercube or A-Hypercube as a function evaluation subroutine include Jarvis' location-allocation problem [13], Berman and Larson's congested median problem [4], Benviniste's location-allocation problem [1]. and Berman, Larson and Parkan's stochastic queue p-median problem [3]. These methods are heuristic local improvement approaches that assume it is possible to locate a vehicle in every zone.

2.2.1 Mean Service Calibration

Call location-dependent service time can be modeled using the Mean Service Calibration method (denoted MSC). As in cases of Jarvis [13] and Halpern [12] where mean service time, as opposed to the distribution of service time, and show that is sufficient to obtain accurate estimates of system performance. The major shortcoming of MSC is that either Hypercube or A-Hypercube is evaluated in each iteration; it can thus be a computationally expensive approach.

To eliminate the computational inefficiency of the MSC method, Jarvis [14] developed an approximation model for spatially distributed queueing systems [1]. The model assumes that call service time is call location-dependent, where all vehicles have the same service rate and utilization while service is exponentially distributed.

2.2.2 Calibration Process

In certain EMSs and other emergency systems, travel times may represent a considerable part of service times. In such cases, it may be advisable to adjust the service times by means of a calibration process, which can be performed using a simple iterative procedure that is proposed by Berman et al. [3]. Basically, the procedure consists of verifying if there are significant differences among the input mean service times and the output mean service times (computed by the hypercube model). In this case, the hypercube is solved using the computed mean service times as inputs, until the differences among input and output values are sufficiently small. This procedure is called a calibration process. Note that it takes into account that the mean travel time depends on the location of the user and the identity of the server. Empirical experiments show that this procedure usually converges in two or three iterations, for a reasonably accurate estimation of the mean service times, although

[1] See Appendix A for more infomation

a formal proof of the convergence of the method is apparently not available in the literature.

2.3 Simulation Models

Simulation models can be formulated with great detail and hence can be validated.

Simulation is used for evaluating EMS system performance in numerous papers as in Savas [19], Berlin and Liebman [2], and Swoveland et al. [20],

In general, simulation models provide several measurement outputs, a drawback is that they are rarely used because of high runtime and data collection costs. These models have some questionable assumptions, but successful applications do exist in the literature.

Chapter 3

Framework

3.1 Problem Statement

The location and dispatching policies used by insurance agencies should aim to arrive to the accident areas as early as possible, due to several reasons such as:

1. providing a timely service to their customers
2. helping clear out the accident area
3. keeping the workloads of its adjusters as balanced as possible

The location policies should be optimized for service time but they should also consider cooperation based on adjuster workload. However, current insurance agency policies are empirical, and do not consider cooperation. By neglecting this, adjusters tend to take more time to arrive at the accident site, making the insurance agency less competitive, and generating more traffic congestion.

The financial costs of applying an empirical policies instead of optimum policies is difficult to measure. However the costs include more use of fuel, more use of vehicles that implies more maintenance costs, and the opportunity cost of losing a customer for low quality-of-service. The use of quantitative models may also help in what-if analysis to assess the overall service rate if more adjusters are placed. The use of mathematical models to determine better location policies based on scenarios, and the use of real data to simulate and evaluate new scenarios versus the current policy is one of the main contribution on this work.

The benefit of this framework is that several policies can be evaluated for different scenarios (for instance high level of congestion in rainy days), and the best policy for each scenario can be determined.

In summary the location of adjusters could be improved with the use of mathematical models and simulation, and obtain several benefits.

The problem studied in this thesis consist of given a number of adjusters, a set of potential site for place them (**basis**), and a set of demand points, we have to determine where to place the adjusters, so as to minimize the average response time, assuming that calls arrive with a Poisson distribution and with an own arrival rate for each demand point.

3.2 Mathematical Framework

Two mathematical models are proposed, the first model (model A) was created base on the one proposed by Goldberg [10] for which we made some relaxations instead for obtaining a linear model.

The second model (model B) has additional simplification, considering that it is unlikely to assign an adjuster to a demand point covered previously by more adjusters, omitting allocation variables and adding restrictions to guarantee the correct order of allocation is introduced.

3.3 Model A

This model is based on the model proposed by Goldberg [10], and it contains assignment variables for all possible orders.

The following assumptions are considered in the model:

- The probability that an adjuster is busy is ρ and is unaffected by the state of the system.
- There is a strict ordering of the basis preferred for each zone that does not depend on the current state of the system.
- All calls are answered by an adjuster originating from its base, not in route back to the base.
- The arrival of calls to the system follows a stationary distribution.
- The model is presented using a 0-queue assumption.

Sets and indexes:

- n number of demand points
- m number of potential sites to locate adjusters/facilities
- p number of available adjusters
- i index for demand points; $i \in V = \{1, 2, \ldots, n\}$
- j index for potential site for adjusters/facilities $j \in W = \{1, 2, \ldots, m\}$
- k index for possible order; $k \in K = \{1, 2 \ldots, p\}$
- $S_{ij} = \{r \in W |$ site r is preferred by proximity before site j for demand point $i\}$

Parameters:

- λ_i arrival rate of calls for demand point i
- ρ is the utilization of each adjuster, the value is between 0 and 1, where 0 means that the server is always idle. To obtain an approximate value for ρ we use the formula proposed by Berman et al. [4] $\rho = \frac{\sum_{i=0}^{n} \lambda_i}{mp}$
- t_{ij} is the expected travel time between demand point i and potential site j.
- h_{ij}^k is the probability that adjuster j serves point i given that it is the k-th preferred. It is calculated using the following formula: $\mathrm{h}_{ij}^k = (1-\rho)\rho^{k-1}$

Variables:

- $x_j = \begin{cases} 1 & \text{if an adjuster is placed at potential site } j \\ 0 & \text{otherwise} \end{cases}$
- $y_{ij}^k = \begin{cases} 1 & \text{if the adjuster at site } j\text{, is the } k\text{-th to cover demand point } i \\ 0 & \text{otherwise} \end{cases}$

Model

$$\min \sum_{j=1}^{m} \sum_{k=1}^{p} \sum_{i=1}^{n} h_{ij}^k t_{ij} y_{ij}^k \tag{3.1}$$

Minimize the average expected response time subject to

$$\sum_{j \in W} x_j = p \tag{3.2}$$

Only locate p adjusters

$$\sum_{j \in W} y_{ij}^k = 1 \qquad i \in V, k \in K \tag{3.3}$$

Each demand point i is covered by an adjuster on each order k

$$y_{ij}^k \leq x_j \qquad i \in V, j \in W, k \in K \tag{3.4}$$

Relationship between variables x and y

$$\sum_{k=1}^{p} y_{ij}^{k} \leq x_j \qquad i \in V, j \in W \tag{3.5}$$

For each located adjuster, there can only be a maximum of one ordered assignment.

$$y_{ij}^{k} \leq \sum_{r \in S_{ij}} y_{ir}^{k-1} \qquad i \in V, j \in W, k \in K \setminus \{1\} \tag{3.6}$$

Assign j to cover i in order k only if the assignment of order $k-1$ was made for some $r \in S_{ij}$

$$x_j \in \{0,1\} \qquad j \in W$$
$$y_{ij}^{k} \in \{0,1\} \qquad i \in V, j \in W, k \in K$$

Observe that we do not need to add a constraint to ensure the counterpart of (3.5) because (3.2) and (3.4) ensure that each adjuster must cover each demand point for some order, therefore if an adjuster located at j does not cover demand point i at order k (indicated by the maximum covering order in S_{ij}) there will be at least one adjuster that does not cover demand point i at any order resulting in an infeasible solution.

3.4 Model B

This model was developed with the idea that it is unlikely that the farthest adjusters serve demand points on cases where the system does not become congested. In these cases we can make the assumption that the probability of being served by the ℓ-th adjuster is almost zero, where ℓ is large enough but less than p.

Parameters:

- M is a large integer
- ℓ the number of allowed adjusters per demand point

- a_{ik} the k-th preferred location server regarding the point i.

Variables:

- z_j the number of adjusters placed at site j
- y_{ij}^k if adjuster in j, is the k-th to cover demand point i

The objective, and constraints (3.2)-(3.5) are practically the same as in model A, with the difference that binary variables x_j from Model A are replaced by integer variables z_j inspired by the results of Berman [3], and the addition of the following binary variables

- $u_{ij} = \begin{cases} 1 & \text{if the number of adjusters between } i \text{ and } j\text{, inclusive, is less than } \ell \\ 0 & \text{otherwise} \end{cases}$
- $v_{ij} = \begin{cases} 1 & \text{if the number of adjusters between } i \text{ and } j\text{, is less than } \ell - 1 \\ 0 & \text{otherwise} \end{cases}$

Model

$$\min \sum_{j=1}^{m} \sum_{k=1}^{\ell} \sum_{i=1}^{n} h_{ij}^k t_{ij} y_{ij}^k \tag{3.7}$$

Minimize the average expected response time

$$\sum_{j \in W} z_j = p \tag{3.8}$$

Only locate p adjusters

$$\sum_{j \in W} y_{ij}^k = 1 \qquad i \in V, k \in \{1, \dots, \ell\} \tag{3.9}$$

Each demand point i is covered by an adjuster on each order until ℓ

$$y_{ij}^k \le z_j \qquad i \in V, j \in W, k \in \{1, \dots, \ell\} \tag{3.10}$$

Relationship between variables z and y

$$\sum_{r\in S_{ij}\cup\{j\}} z_r + (p-\ell)u_{ij} \leq p \qquad i\in V, j\in W \qquad (3.11)$$

$$\sum_{r\in S_{ij}\cup\{j\}} z_r + Mu_{ij} \geq \ell + 1 \qquad i\in V, j\in W \qquad (3.12)$$

These two constraints set the relationship between the z and u variables. If $u = 1$ (3.12) become redundant, and contraint (3.11) guarantees that the number of adjusters between i and j is less or equal than ℓ, otherwise if $u = 0$ the equation (3.11) become redundant, and equation (3.12) guarantees that the number of adjusters between i and j is more than ℓ.

$$\sum_{k=1}^{\ell} y_{ij}^k + M(1-u_{ij}) \geq z_j \qquad i\in V, j\in W \qquad (3.13)$$

Assign z_j times j to i if $u_{ij} = 1$, otherwise if becomes redunant.

$$\sum_{r\in S_{ij}} z_r + (p-(\ell-1))v_{ij} \leq p \qquad i\in V, j\in W \qquad (3.14)$$

$$\sum_{r\in S_{ij}} z_r + Mv_{ij} \geq \ell \qquad i\in V, j\in W \qquad (3.15)$$

Analogus to constraints (3.11)-(3.12) these constraints set the relationship between the z and v variables.

$$\sum_{k=1}^{\ell} y_{ij}^k + M(1-v_{ij}+u_{ij}) \geq \ell - \sum_{r\in Sij} z_r \qquad i\in V, j\in W \qquad (3.16)$$

$$\sum_{k=1}^{\ell} y_{ij}^k - M(1-v_{ij}+u_{ij}) \leq \ell - \sum_{r\in S_{ij}} z_r \qquad i\in V, j\in W \qquad (3.17)$$

Assign j to i the times remaining to complete ℓ assignments

$$y_{ij}^k \leq u_{ij} + v_{ij} \qquad i\in V, j\in W \qquad (3.18)$$

Assign j to i only if it is in the first ℓ adjusters near i

$$z_j \in \{0,1,\ldots,p\} \qquad j\in V$$

$$y_{ij}^k \in \{0,1\} \qquad i\in V, j\in W, k\in I$$

$$u_{ij}, v_{ij} \in \{0,1\} \qquad i\in V, j\in W$$

3.5 COMPARISON

As can be seen, model A has a considerable amount of binary variables, therefore model B was formulated. However, due the lack of allocation variables we need to add more variables and constraints to ensure a similar behavior.

The size of the models as a function of N, M, p and ℓ is shown in table 3.5

	Model A	Model B
variables	$m(np+1)$	$m(n(\ell+2)+1)$
constraints	$n(2mp+p)+1$	$n((l+8)m+1)+1$

Table 3.1: Models size

Since $\ell < p-2$ it can noticed that model B has lees variables than model A, and when $\ell < 2p-8$ model B has less constrains too.

Chapter 4

Heuristic Procedures

Because the problem stochasticity can not evaluate solutions accurately, so the Hypercube model is used in conjunction with the service mean time calibration process (see Berman et al. [3]) to obtain the main parameters.

4.1 Proposed Metaheuristic

A Scatter-Search with a dynamic reference set was implemented, and a path relinking method is used as combination method. We used a dynamic reference set given that the number of solutions generated in each combination is large. To enhanced the solutions two improvement methods are included, one proposed by Berman [3], and a proposed Local Search.

In Algorithm 1 (see below) a Reference Set is created, consisting in the bests b solutions, and the most diverse b solutions. To measure the diversity of a solution, the minimum cost perfect matching with each of the solutions of the Reference Set.

In Algorithm 2 (see bellow), each pair of solutions is combined using path relinking. However, since the Reference Set is dynamic some solutions are displaced before been combined.

Algorithm 1 Scatter Search

procedure INITIAL PHASE
    $Sols \leftarrow SeedSolutions()$
    repeat
        $DiversificationGenerator(Sols)$
        $Improvement(Sols)$
        $ReferenceSet(Sols)$
    until $HasNotChanged(Sols)$ **or** $MaxIterations$
end procedure

Algorithm 2 Scatter Search

procedure SCATTER SEARCH PHASE($RefSet$)
    repeat
        $G_X \leftarrow SubsetGeneration(RefSet)$
        for all $X \in G_X$ **do**
            $C_X = SolutionCombination(X)$
            for all $Sol \in C_X$ **do**
                $Improvement(Sol)$
                $Update(RefSet, Sol)$
            end for
        end for
    until $HasNotChanged(RefSet)$ **or** $MaxIterations$
end procedure

4.2 Description of Components

The components of Scatter Search template [9] consist of specific subroutines of the following types:

- A Diversification Generator
- An Improvement Method
- A Reference Set Update Method
- A Subset Generation Method

We described below each of the components.

4.2.1 A Diversification Generator

To generate a collection of diverse trial solutions, using an arbitrary trial solution (or seed solution) as an input. In our case, the generator was a multi-start consisting of a GRASP

4.2.1.1 GRASP

We choose GRASP because in the proposed problem any allocation of sites for adjusters is a feasible solution, so it was decided to give some intelligence to this simple allocation instead of choose random points. We start with a partial solution (location of a smaller number of adjusters) with only one adjuster allocated, evaluate each site with the greedy function, and choose one from the best α evaluations, until each adjuster was locate. Several functions were evaluated for constructive algorithm but because presented cooperativeness it is difficult to approximate the final results

Algorithm 3 Path Relinking Initial Phase

procedure PathRelinking($Instance$)

$EliteSols \leftarrow MultiStart(Instance)$

repeat

$MiscSols \leftarrow GenerateMiscSols(Instance, EliteSols)$

for all $x \in MiscSols$ **do**

$ImprovementMethod(x)$

$EliteSols.Update(x)$

end for

until $PerfectMatchingCost(MiscSols, EliteSols) < \epsilon$

$SubsetControl(EliteSols)$

end procedure

in a partial solution with a greedy function. We use three greedy functions to test the construction. The first function, was the p-mean or the sum the distances of each allocation from each located adjuster with their nearest demand points.

$$\sum_{j=1}^{m}\sum_{i=1}^{n} t_{ij}y_{ij}^{1} \tag{4.1}$$

For the second function, we try to incorporate cooperatively, including in the evaluation the distance of allocations from each located adjuster, with their k nearest demand points plus *idle* probability given that is allocated to their $k-1$ nearest demand points.

$$\sum_{j=1}^{m}\sum_{k=1}^{p}\sum_{i=1}^{n} h_{ij}^{k}t_{ij}y_{ij}^{k} \tag{4.2}$$

The third function, consist in use the Mean Service Time (MST) calibration method proposed by Jarvis [14], to obtain more accuracy values of the current mean response time.

4.2.2 An Improvement Method

To transform a trial solution into one or more enhanced trial solutions. (If no improvement of the input trial solution results, the "enhanced" solution is considered to be the same as the input solution.)

We have two improvement methods, the first an adaptation of the proposed method by Berman et al. [3] and the second, a Local Search described in the next section.

4.2.2.1 Local Search

Due to the cost of assessing each of the different solutions, it was decided to design a local search to only evaluate a small neighborhood. The movement consist in remove the server with less workload, and place it near the server with more workload. Looking for a better balance workloads.

4.2.3 A Reference Set Update Method

To build and maintain a Reference Set consisting of the b best solutions found (where the value of b is typically small, e.g., between 20 and 40), organized to provide efficient accessing by other parts of the method.

We choose a two tier Reference Set to maintain a part of diverse solutions, this because the combination method generates similar solutions to input. And we opt for be dynamic since the number of generated solutions is big (proportional to p).

Whenever the Reference Set is update it is try to incorporate the solutions generated in a combination, because the Reference Set is divided in two parts, first try to accommodate each solution by quality criteria. If a solutions enters the

Reference Set, degrades solutions lower quality to it, removing the worst solution of the Reference Set.

After trying to accommodate solutions for quality criterion, solutions that do not enter the Reference Set and those that are removed, they are try to enter with diversity criterion. reassessing the diverse solutions, since they evaluation depends on the quality members.

4.2.4 A Subset Generation Method

To operate on the Reference Set, to produce a subset of its solutions as a basis for creating combined solutions. We only look for subsets of size two i.e. pairs of solutions, because our Reference Set is dynamic, to identify the new solutions, we label solutions as new and old at the start of each iteration. We combine first the new solutions between them, next the new solutions with the old, the new solutions that enters in the Reference Set as a combined solutions, is not part of the subsets until the next iteration. Each solution generated and did not enter in the Reference Set, displaced by a better solution, or actually a member of the diverse part of the Reference Set is evaluated to be in the Reference Set as a diverse solution.

4.2.5 A Solution Combination Method

To transform a given subset of solutions produced by the Subset Generation Method into one or more combined solution vectors.

We choose a path relinking as a combination method consisting of determine a match between servers, this match can be

- Perfect Matching: minimizing the distance between paired servers

Algorithm 4 Subsets Generator

```
procedure GenerateSubsets(RefSet, NowTime)
    NewSols ← SolutionsSince(RefSet, NowTime)
    for all (Sol_x, Sol_y) ∈ NewSols do
        if Sol_x ∈ RefSet and Sol_y ∈ RefSet then
            CombinedSols ← PathRelinkingCombination(Sol_x, Sol_y)
            Update(RefSet, CombinedSols)
        end if
    end for
    UpdateDiversity(RefSet)
    if NumberOfOldSols(RefSet, NowTime) > 0 then
        OldSols ← SolutionsUntil(RefSet, NowTime)
        for all Sol_x ∈ NewSols do
            for all Sol_y ∈ OldSols do
                if Sol_x ∈ RefSet and Sol_y ∈ RefSet then
                    CombinedSols ← PathRelinkingCombination(Sol_x, Sol_y)
                    Update(RefSet, CombinedSols)
                end if
            end for
            UpdateDiversity(RefSet)
        end for
    end if
end procedure
```

- Workload Matching: sorting the servers of both solutions according to the workloads, and match them according these sorted lists.

- Random Matching

once we have the matching we proceed from one solution to interchange each server (one by step) until end in the other solution, in each interchange we have a new solution. To make the interchanges we have three options

- Nearest First

- Farthest First

- Random

Algorithm 5 Path Relinking Combination Method

```
procedure PAHTRELINKINGCOMBINATION(Sol_x, Sol_y)
    CombinedSols ← EmptyList()
    match ← Matching(Sol_x, Sol_y)                    ▷ perfect,workload,random
    order ← ProcessOrder(Sol_x, match, Sol_y)         ▷ nearest/farthest first,random
    for i ← 1, p do
        j ← order[i]
        if Sol_x.ServerLocation(j)! = Sol_y.ServerLocation(match[j]) then
            Sol_x.SetServerLocation(j, Sol_y.ServerLocation(match[j]))
            CombinedSols.insert(Sol_x)
        end if
    end for
end procedure
```

Chapter 5

Computational Experiments

In this chapter computational experimentation occurs held to validate model results, and adjust the parameters of the proposed heuristic.

5.1 Models

These test were evaluated in a Lanix Spine BW Processor Intel Xenon, CPU E5-2867W, 3.10 GHz. With operative system Ubuntu 14.04.3 LTS The models were solved with CPLEX 12.6.0.0, and they were coded in C++.

Different random instances were generated to validate the proposed formulations. For $n = 100$ we created 8 instances with $m = 20, 30, \ldots, 100$, and they were tested with different values of p from 5 to $\frac{m}{2}$, shown in the following table 5.1.

Model A was solved to optimality only when $p < 25$, in an average time of 217 seconds. for the rest of the values of p, no factible solution was found for 65% of the test cases in less than 20 minutes, for the rest test cases, an average gap of 27% was obtained.

Model B was evaluated with the same combinations of n, m, p and ℓ from $1, \ldots, p$. For almost all cases, model B was solved to optimality in less than a

n	m	p
100	20	5-10
100	30	5-15
100	40	5-20
100	50	5-25
100	60	5-30
100	70	5-35
100	80	5-40
100	90	5-45
100	100	5-50

Table 5.1: Instances to test models

minute for each case.

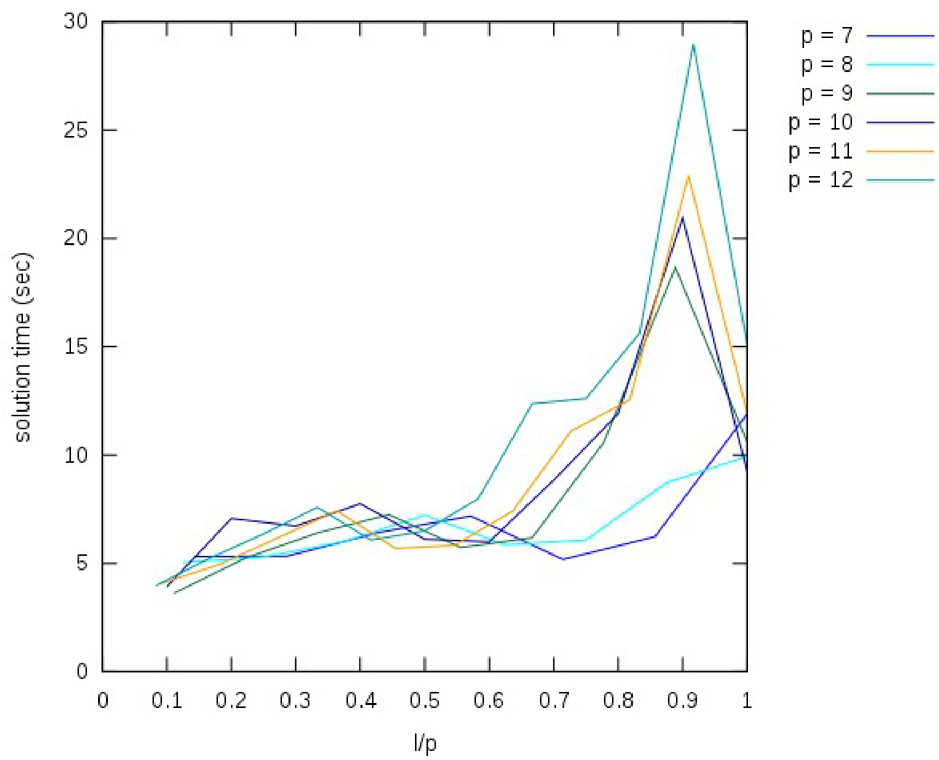


Figure 5.1: Solution times versus ℓ between p

When the value of ℓ is close to p model resolution takes longer and in some cases optimal is not achieved.

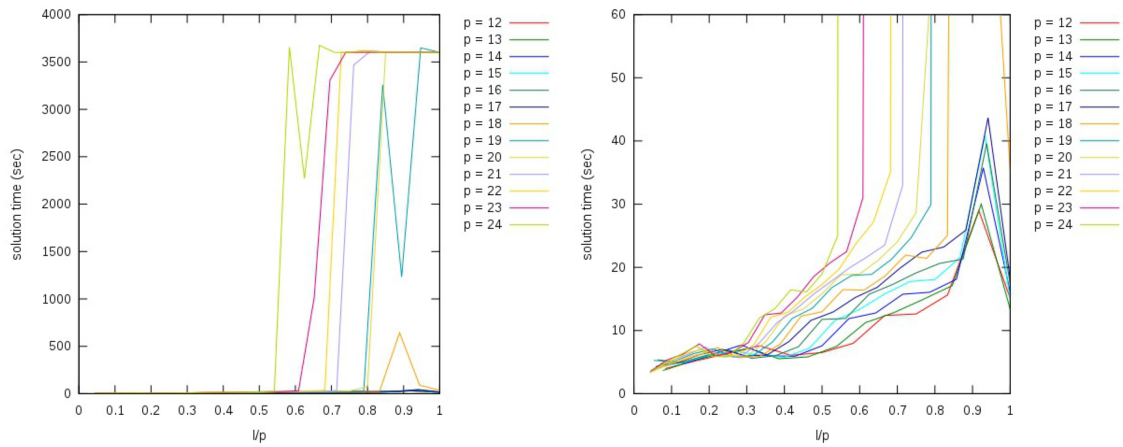
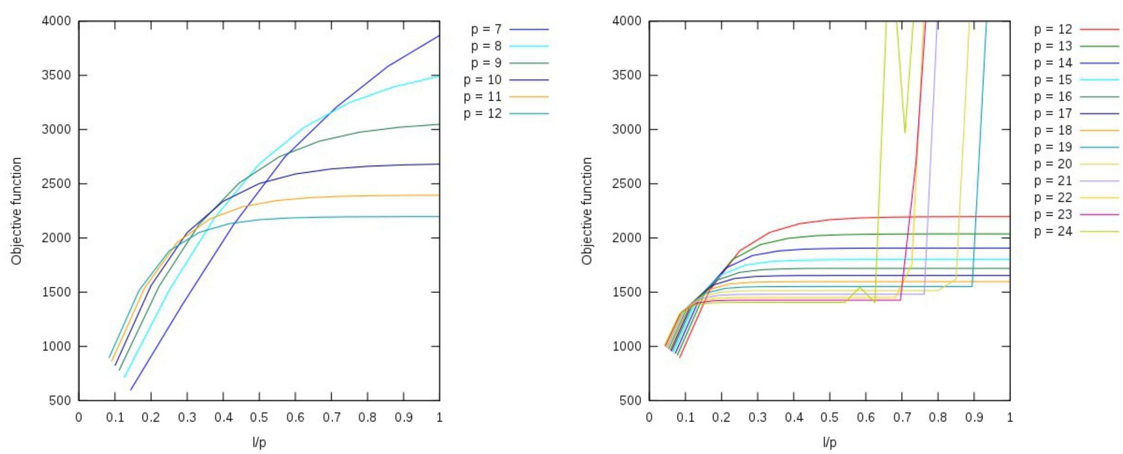


Figure 5.2: Solution times versus ℓ between p

However, it can be seen that for different values of ℓ the solutions obtained are the same.



5.2 GRASP

The purpose of this experiment is to assess the behavior of the three greedy functions used by the greedy randomized construction of the diversificatione genrator method, as well as the α parameter, To this end we run the heuristic consiting of the construction phase only (that is ousidide of the Scatter Search - Path Relinking method). The results are shown in table **??**.

Several instances were tested with different α values (from 0 to 1 in steps of 0.05) to determine the value of α to use. The value of $\alpha = 0$ means that the procedure becomes completely greedy, and a value of $\alpha = 1$ means that the procedure becomes completely random.

The test suggest that with a more random version there are more different solutions and a wide range of objective values achieving better solutions.

5.3 Path Relinking

As we mentioned before, the path relinking algorithm consist of two key components the match method and the processing method.

This experiment aims to determine which methods are best suited. Measuring the relative improvements achieved regarding the best solution obtained with the multi-start.

	M	50			100			150		
match	order	30	50	75	30	50	75	30	50	75
perfect	farthest first	5.18	10.99	15.94	6.26	10.11	11.55	5.14	7.93	10.56
	nearest first	5.22	11.30	14.43	5.61	9.15	10.70	5.26	6.73	10.01
	random	5.58	9.45	12.08	5.23	7.61	10.65	4.02	6.13	8.14
random	farthest first	first	9.96	12.04	4.63	8.14	10.72	3.33	6.59	8.60
	nearest first	6.90	11.36	16.02	6.13	10.16	12.95	5.82	9.43	10.42
	random	5.20	9.73	13.97	4.68	7.70	10.24	4.24	7.18	9.20
workload	farthest first	5.06	7.31	9.68	4.43	5.73	7.13	4.59	5.86	7.01
	nearest first	5.17	10.93	13.67	4.87	9.43	11.02	5.04	7.96	9.07
	random	4.58	7.81	10.11	3.34	6.39	6.89	3.48	6.16	6.61

Table 5.2: Path Relinking Results

5.4 Scatter Search

For this case we are interested in knowing as the heuristic contributes to improvement, at par, we want to know how much time is spent calling the Hypercube.

The improvement is obtained between the value of the best solution generated on the multi-start, and the best solution after the SS-PR without improvement methods.

To evaluate this heuristic several instances was tested and a design of experiment was made to determine the match method and the processing order method that produce better results.

M		50			100		
N	p	Improvement (%)	Time (sec)	MST(sec)	Improvement (%)	Time (sec)	MST(sec)
30	7	6.97	1.64	1.55	6.70	4.33	4.10
	10	9.62	5.20	4.57	5.74	9.97	9.65
	15	6.95	8.55	6.49	3.38	14.90	13.46
	20	3.67	13.79	8.80	2.80	28.30	24.01
50	7	9.76	2.47	2.41	7.68	4.23	4.46
	10	12.20	7.09	6.15	8.09	11.01	10.59
	15	9.75	11.35	8.51	9.55	25.72	22.36
	20	10.63	30.22	19.38	9.51	61.22	49.41
75	7	11.64	2.43	2.37	7.70	4.55	4.76
	10	14.50	6.16	5.35	11.46	10.17	9.82
	15	18.26	19.44	14.08	11.95	36.70	31.19
	20	13.59	44.19	28.05			
		10.63	12.71	8.98	7.69	19.17	16.73

Table 5.3: Scatter Search improvements and time

Chapter 6

Conclusions

- Two different models (Model A and Model B) for the car wreck adjusters location have been proposed. The models can be seen as linear approximations to the SQpM.
- We observed that Model A has difficult solving medium size instances.
- For small enough values of ℓ, Model B can be solved relatively quick for instances of medium size.
- The solutions obtained from the Model B for small values of ℓ approximates well the solutions of Model A, as well as self solutions for large values of ℓ

6.1 Future Work

Due to policy changes of the insurance agency that collaboration had, It was not possible to obtain information to create real instances, and measure the benefits of the methods here proposed, against the policies currently used.

Some tests are pending to be done

- Extend the testing of models

- Comparison between simulation results and Mean Service Time Calibration Process for diverse solutions.

Appendix A

Approximating the Equilibrium Behavior of MSLS

In Jarvis [14] a procedure is given for approximating the equilibrium behavior of multi-server loss systems having distinguishable servers and multiple customers types under light to moderate traffic intensity.

A.1 Introduction

In an emergency service such as fire or police, the servers are fire fighting units or patrol cars and the customers are calls for service. The simple Erlang loss system is inadequate in two aspects for a detailed system analysis.

- one often wishes to preserve the identity of service units (distinguishable servers).
- because of the geographic nature of these systems, the service time depend on both the server and the customer at least through the travel time between the pair.

A.2 Model assumptions, Notation, and Terminology

Consider a system in which:

- Exactly one server is assigned to each customer unless all servers are busy, in which case the customer is irrevocably lost from the system
- Servers are assigned to customers according to a fixed preference assignment rule
- No preemption of service is allowed
- Assignments are made immediately upon customer arrival

and we have the following parameters

- N distinguishable servers
- C types of customers
- Customers of type m arrive according to a Poisson process with rate λ_m
- λ total arrival rate
- a_{mk} be the kth preferred server for customers of type m
- τ_{im} the expected service time for server i and customer of type m

The performance measures for the system include

- ρ_i the workload of server i
- f_{im} the probability a random customer of type m is assigned to server i
- P_N the probability all servers are busy

A.3 Approximation Procedure

The procedure described below is based on that given by Larson [16], for approximating performance measures for the Hypercube model assuming exponential service times. Larson developed an approximation for f_{im} as

$$f_{im} \simeq Q(N, p, k-1)(1-\rho_i)\prod_{l=1}^{k-1}\rho_{a_{ml}} \tag{A.1}$$

where

$$Q(N, p, k) = \sum_{j=k}^{N-1} \frac{(N-j)(N^j)(\rho^{j-k})P_0(N-k-1)!}{(j-k)!(1-P_N)^k N!(1-\rho(1-P_N))} \text{for} k = 0, 1, \ldots, N-1 \tag{A.2}$$

Let B_i denote the event that server i is busy; and let B_{im} denote the event that server i is busy serving a customer of type m, then

$$\rho_i = Pr\left[B_i\right] = \sum_{m=1}^{C} Pr\left[B_{im}\right] = \sum_{m=1}^{C} \lambda_m f_{im}\tau_{im} \tag{A.3}$$

combine equations (A.1) and (A.3) and solve for ρ_i to obtain the approximation iteration

$$\rho_i(\text{new}) = \frac{V_i}{(1+V_i)} \tag{A.4}$$

where V_i is given by

$$V_i = \sum_{k=1}^{N} \sum_{m:a_{mk}=i} \lambda_m \tau_{im} Q(N, \rho, k-1)\prod_{l=1}^{k-1}\rho_{a_{ml}} \tag{A.5}$$

When there is a common mean service time, the estimates for ρ_i can be normalized using

$$\sum_{i=1}^{N} \rho_i = N\rho(1-P_N) \tag{A.6}$$

In the generalized procedure, τ can be approximated at the end of each iteration by

$$\tau = \sum_{m=1}^{C}\left(\frac{\lambda_m}{\lambda}\right)\sum_{i=1}^{N}\frac{\tau_{im} f_{im}}{(1-P_N)} \tag{A.7}$$

Approximation Algorithm

Given:

$$\lambda_m, \tau_{im}, a_{mk} \text{ for } m = 1, \ldots, C; i = 1, \ldots, N; k = 1, \ldots, N$$

Initialize:

$$\rho_i = \sum_{m: a_{m1} = i} \lambda_m \tau_{im}; \ \tau = \sum_{m=1}^{C} (\lambda_m / \lambda) \tau_{a_{m1}, m}$$

Iteration:

(1) Compute $Q(N, \rho, k)$ for $k = 1, \ldots, N-1$ where $\rho = \lambda\tau/N$ using equation (A.2).

(2) For $i = 1, \ldots, N$, the new ρ_i is $V_i/(1 + V_i)$, where V_i is given by equation (A.5).

(3) Stop if max change in ρ_i is less than convergence criterion.

(4) Else compute P_N by equation (A.6), τ by equation (A.7), and f_{im} by equation (A.1).

(5) Return to step 1.

No analytic bounds on the accuracy or convergence properties of the approximation procedure have been developed to the best of my knowledge.

In regards to convergence properties, the numerical iteration has proved to be very stable and converges in a small number of iterations under relatively stringent conditions, with 4 to 6 iterations being typical for 10-servers systems.

In comparing the accuracy of this approximation to results of the exact Hypercube model, Larson has found errors in server workloads to be less than 1 to 2 percent.

Appendix B

Berman Heuristic

This problem is presented by Berman et al. [3] and describes a similar situation to our case, with the difference that the location of servers is in all the network, including the edges.

Also present two easily programmable heuristics to solve the proposed problem. And uses the **Mean Service Time Calibration Process** to evaluate the solutions.

B.1 The Stochastic Queue p-Median Problem

To define the problem the following definitions are made

- $G(N, L)$ the transportation network
- N the set of demand centers, with $|N| = n$
- L the set of all transportation arteries, the links
- h_j the fraction of service calls associated with each node j
- $d(X, Y)$ is the shortest path between any two points $X, Y \in G$
- p number of response units

- $\bar{X}$ the home locations of the service units while available
- λ mean rate per unit of time within service calls are generated in Poisson manner

Given the arrival of a call for service, exactly one of the servers is dispatched to it assuming that at least one server is available.

The service time for any service unit i is the sum of two components:

- The non-travel time component, which is the sum of on-scene and off-scene service time.
- Travel time component, which is the sum of travel time to the location of the call and travel time back to the home location.

The mean service time for a service unit located at $\bar{X}^i$ is denoted $S(\bar{X}^i)$,

$$S(\bar{X}^i) = \sum_{j=1}^{n} h_j^i \left(\bar{W}_{ij} + \frac{\beta_i}{v_i} d(\bar{X}^i, j) \right) \; i = 1, \ldots, p \tag{B.1}$$

- $\bar{W}_{ij}$ is the mean of the **non-travel time** component W_{ij}
- v_i is the **travel speed** of unit i to the scene of the call which is assumed constant
- β_i is a constant that allows different travel speeds to and from the scene of the call
- h_j^i is the probability that server i is dispatched to node j given that server i is dispatched to a call for service.

Whenever a call for service arrives while at least one of the servers is free at its home location, the closest available server to the call will be dispatched. Calls that find all servers busy enter a queue. The queue discipline is assumed to be

First-Come-First-Served. The expected response time to a random call denoted by $\bar{T}_R(X)$ is the sum of two components

$$\bar{T}_R(\bar{X}) = \bar{W}_q(\bar{X}) + \bar{t}(\bar{X})$$

- $\bar{W}_q(\bar{X})$ is the expected **waiting time** in the queue
- $\bar{t}(\bar{X})$ is the expected **travel time** to the call.

The objective is to find a set of p locations $\bar{X}^*$ on the network such that

$$\bar{T}_R(\bar{X}^*) \leq \bar{T}_R(\bar{X}) \; \forall \bar{X} \in G$$

$\bar{X}^*$ is called **the stochastic queue p-median**.

B.2 Mean Service Time Calibration Process

In this emergency system, travel times may represent a considerable part of service times. It may be advisable to adjust the service times by means of a calibration process, which can be performed using a simple iterative procedure.

The procedure consists of verifying if there are significant differences among the input mean service times and the output mean service times (computed by the hypercube model). In this case, the hypercube is solved using the computed mean service times as inputs, until the differences among input and output values are sufficiently small.

The mean service time calibration method

Step 0. The mean service time of unit i, $1/\mu^i = 1/\mu^i_{NT}$, $i = 1, \ldots, p$ ($1/\mu^i_{NT} = \sum_{j=1}^{n} h_j \bar{W}_{ij}$ is the mean of non-travel time component of the service time).
Step 1. Run the Hypercube Model (using μ^i) to obtain f_{ij}, $i = 1, \ldots, p$, $j = 1, \ldots, n$.
Step 2. $1/\hat{\mu}^i = \sum_{k=1}^{n} h^i_k (\bar{W}_{ik} + (\beta_i/v_i) d_{ik})$ where $h^i_k = f_{ik} / \sum_{j=1}^{n} f_{ij}$

Step 3. If $|1/\hat{\mu}^i - 1/\mu^i| > \epsilon$ for at least one i, $i = 1, \ldots, p$, set $1/\mu^i \equiv 1/\hat{\mu}^i$ and go back to *Step 1.* Otherwise stop.

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